- Sequence and Series

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AP Ex $\quad a_1 \quad a_2 \quad a_3 \quad \cdots\cdots \quad a_n$

$a, \; a+d, \; a+2d \quad \cdots\cdots \quad a+(n-1)d$

$d = a_r - a_{r-1}$

$a_n = a+(n-1)d$

Ex $2, 5, 8, 11, \ldots\ldots$

$a_2 - a_1 = 3$

$a_3 - a_2 = 3$

$\vdots \quad d = 3$

(AP)

$S_n = a + (a+d) + (a+2d) \cdots\cdots + a+(n-1)d \quad —①$

$S_n = a+(n-1)d + a+(n-2)d + a+(n-3)d + \cdots + a \quad —②$

Ex $\sqrt{3}, 2\sqrt{3}, 3\sqrt{3}, 4\sqrt{3}, \ldots\ldots$

$d = \sqrt{3}$

① + ②

$$2S_n = \underbrace{2a+(n-1)d + 2a+(n-1)d + \cdots\cdots + 2a+(n-1)d}_{n \text{ terms}}$$

$$2S_n = (2a+(n-1)d) \times n$$

$$\boxed{S_n = \frac{n}{2}\left(2a+(n-1)d\right)}$$

$\frac{n}{2}\left(a + a+(n-1)d\right)$

$$\boxed{= \frac{n}{2}\left(a_1 + a_n\right)}$$

In General

$\frac{n}{2}\left(a+d + a+(n-2)d\right)$

$\frac{n}{2}\left(a_2 + a_{n-1}\right)$

$\frac{n}{2}\left(a_r + a_{n-r+1}\right)$

r^{th} term from the start

r^{th} term from the end

Airthematic mean $\cdots\cdots$ Find a number 'b' b/w a & c such that a, b, c are in AP

$a_1 = a$

$a_2 = b$

$a_3 = c$

$d = a_2 - a_1 = b - a$

$d = a_3 - a_2 = c - b$

$b - a = c - b$

$2b = a + c$

$$\boxed{b = \frac{a+c}{2}}$$

AM

$2, \; \underline{7}, \; 12$

$5 \quad 5$

$\frac{2+12}{2} = 7$

$\boxed{\begin{matrix} a = 2 \\ d = 7 \end{matrix}}$

★ Insert 'n' terms b/w a & b such that all the terms are in AP

$a, A_1, A_2, \ldots\ldots, A_n, b$
$a_1, a_2, a_3 \qquad\qquad , a_{n+1}, a_{n+2}$

$a_1 = a \quad -①$

$a_{n+2} = b \Rightarrow a_1 + (n+2-1)d = b$ ↓ from ①
$a + (n+1)d = b$

$$\boxed{d = \frac{b-a}{n+1}}$$

$a_n = a + (n-1)d$

$a_3 = A_2 = a + 2d$
Third term — Second Inserted term

$= a + 2\left(\frac{b-a}{n+1}\right)$

Check...

$a_{n+2} = a + (n+1)d$
$= a + \cancel{(n+1)}\frac{(b-a)}{\cancel{(n+1)}}$
$= \cancel{a} + b - \cancel{a}$
$= b$

GP....
$a_1, a_2, a_3, a_4, \ldots, a_n$
$a, ar, ar^2, ar^3 \ldots\ldots ar^{n-1}$

$$\boxed{a_n = ar^{n-1}}$$

Ex.....
$2, 6, 18, 54, \ldots\ldots$
$\frac{a_2}{a_1} = 3 \quad \frac{a_3}{a_2} = 3, \quad \frac{a_4}{a_3} = 3$

$\boxed{a = 2}$ $\boxed{r = 3}$

Ex $a_{25} = ar^{24}$
$= 2(3)^{24}$

$S_n = a + \cancel{ar} + \cancel{ar^2} \ldots\ldots + \cancel{ar^{n-1}} \quad -①$
$rS_n = \quad \cancel{ar} + \cancel{ar^2} \ldots\ldots + \cancel{ar^{n-1}} + ar^n \quad -②$

② − ①

$rS_n - S_n = ar^n - a$
$S_n(r-1) = a(r^n - 1)$
$S_n = \frac{a(r^n-1)}{(r-1)}$ .....

Special Case $-1 < r < 1$, $n \to \infty$ } $\boxed{r^n \to 0}$

$S_n = \frac{a(1-r^n)}{(1-r)} \Rightarrow S_\infty = \frac{a(1-0)}{(1-r)} = \frac{a}{1-r}$

— x — x — x —

Ex....
$1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots \infty$

$\boxed{a = 1 \quad r = \frac{1}{3}}$

$\frac{a_2}{a_1} = \frac{1/3}{1} = \frac{1}{3}$
$\frac{a_3}{a_2} = \frac{1/9}{1/3} = \frac{1}{3}$ .....

$S_\infty = \frac{a}{1-r} = \frac{1}{1-\frac{1}{3}} \Rightarrow \frac{1}{2/3}$

Geometric Mean.....

find 'b' b/w 'a' & 'c' so that a, b, c are in GP.

$a_1 = a$
$a_2 = b$
$a_3 = c$

$r = \frac{a_2}{a_1} = \frac{a_3}{a_2}$ (GP)

$\frac{b}{a} = \frac{c}{b}$

Ex

$$\frac{b}{a} = \frac{c}{b}$$

$$\boxed{b^2 = ac}$$

$$\Rightarrow b = \pm\sqrt{ac}$$

Ex

$$3, \underline{\pm 6}, 12$$

3, 6, 12, 24, 48 — $a=3, r=2$

3, −6, 12, −24, 48 — $a=3, r=-2$

Terms & Sum can fluctuate b/w +ve & −ve

$$S_\infty = \frac{a}{1-r} = \frac{1}{1-\frac{1}{3}} \Rightarrow \frac{1}{2/3} = 3/2$$

—x—x—

Ex

$2, \underline{5}, 8$ $\quad AM = \frac{2+8}{2} = 5$

$2, \underline{4}, 8$ $\quad GM = \sqrt{2\times8} = 4$

$\frac{1}{2}, \underline{\quad}, \frac{1}{8}$ $\quad AM = \frac{\frac{1}{2}+\frac{1}{8}}{2} = \frac{5}{16}$

$\frac{1}{2}, \underline{\frac{1}{4}}, \frac{1}{8}$ $\quad GM = \sqrt{\frac{1}{2}\times\frac{1}{8}} = \sqrt{\frac{1}{16}} = \frac{1}{4}$

Statement → $\boxed{AM \geq GM}$ OR $\boxed{AM - GM \geq 0}$

Proof →

Inserting b/w a & b

a, —, b

$$AM - GM$$

$$= \frac{a+b}{2} - \sqrt{ab}$$

$$= \frac{a+b-2\sqrt{a}\sqrt{b}}{2}$$

$$\Rightarrow \frac{(\sqrt{a})^2 + (\sqrt{b})^2 - 2(\sqrt{a})(\sqrt{b})}{2}$$

$$= \frac{(\sqrt{a}-\sqrt{b})^2}{2} \geq 0$$ (Perfect square) Hence proved.

—x—x—x—

Inserting 'n' terms....

$a, A_1, A_2 \qquad , A_n, b$

$a_1 \qquad a_{n+2}$

$$\boxed{a_n = ar^{n-1}}$$ Remember

$$a_1 = a \quad -(1)$$

$$a_{n+2} = b \quad -(2)$$

$$ar^{n+1} = b$$

$$r^{n+1} = \frac{b}{a}$$

5th term

$$a_5 = A_4 = ar^4$$

5th term of GP — 4th term of GP

$$= a\left(\frac{b}{a}\right)^{\frac{4}{n+1}}$$

Chek $a_{n+2} = ar^{n+1}$

$\gamma^{n+1} = \frac{b}{a}$

$$\boxed{\gamma = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}}$$

$\gamma = \left(\frac{b}{a}\right)^{\frac{1}{3}}$

Chek $a_{n+2} = a\gamma^{n+1}$

$= a\left\{\left(\frac{b}{a}\right)^{\frac{1}{n+1}}\right\}^{n+1}$

$= \cancel{a} \times \frac{b}{\cancel{a}} = b$

Insert 2 term...

$a, \underline{a^{2/3} b^{1/3}}, \underline{a^{1/3} b^{2/3}}, b$

$a_2 = a\gamma = a\left(\frac{b}{a}\right)^{\frac{1}{3}} = a^{2/3} b^{1/3}$

$a_3 = a\gamma^2 = a\left(\frac{b}{a}\right)^{2/3} = a^{1/3} b^{2/3}$

Solve like this...

$a_1 = a$

$a_4 = b \Rightarrow a\gamma^3 = b$

$\gamma^3 = \frac{b}{a}$

$\gamma = \left(\frac{b}{a}\right)^{\frac{1}{3}}$